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A NOTE ON MEASURE OF ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS USING INCOMPLETE BLOCK DESIGNS

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Abstract

Box and Hunter [1] introduced very important concept of rotatability for response surface designs. Measure of rotatability that enable us to assess the degree of rotatability for a given response surface design have been introduced by Park et al. [9]. In this paper, following the method of construction of Park et al. [9], a new measure of rotatability for second order response surface designs using incomplete block designs like pairwise balanced designs (PBD) and symmetrical unequal block arrangements (SUBA) with two unequal block sizes is suggested, which enables us to assess the degree of rotatability for a given response surface design.

1. Introduction

Response surface methodology is a statistical technique that is very useful in design and analysis of scientific experiments. In many experimental situations, the experimenter is concerned with explaining

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certain aspects of a functional relationship $Y = f(x_1, x_2, \ldots, x_v) + \varepsilon$, where Y is the response; $x_1, x_2, \ldots, x_v$ are the levels of v-quantitative variables or factors; and ε is the random error. Response surface methods are useful where several independent variables influence a dependent variable. The independent variables are assumed to be continuous and controlled by the experimenter. The response is assumed to be as random variable. For example, if a chemical engineer wishes to find the temperature (x_1) and pressure (x_2) that maximizes the yield (response) of his process, the observed response Y may be written as a function of the levels of the temperature (x_1) and pressure (x_2) as $Y = f(x_1, x_2) + \varepsilon$.

The concept of rotatability, which is very important in response surface designs, was proposed by Box and Hunter [1]. Das and Narasimham [2] constructed rotatable designs through balanced incomplete block designs (BIBD). Tyagi [8] constructed second order rotatable designs (SORD) using pairwise balanced designs (PBD). Raghavarao [6] constructed SORD using symmetrical unequal block arrangements (SUBA) with two unequal block sizes. If the circumstances are such that exact rotatability is unattainable, it is still a good idea to make the design nearly rotatable. Thus, it is important of know if a particular design is rotatable or, if is not, to know how rotatable it is. Draper and Guttman [3] suggested an index of rotatability. Khuri [5] introduced a measure of rotatability for response surface designs. Draper and Pukelsheim [4] studied another look at rotatability. Park et al. [9] introduced a new measure of rotatability for second order response surface designs and illustrated for 3^k factorial and central composite designs. Victorbabu and Surekha [10] studied a note on measure of rotatability for second order response surface designs by using BIBD. In this paper, following the method of construction of Park et al. [9], a new measure of rotatability for second order response surface designs using incomplete block designs is suggested, which enables us to assess the degree of rotatability for a given response surface design.

2. Conditions for Second Order Rotatable Designs

Suppose we want to use the second order response surface design $D = ((x_{iu}))$ to fit the surface

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i<j}\sum b_{ij} x_{iu} x_{ju} + \varepsilon_u, \tag{2.1}$$

where x_{iu} denotes the level of the i-th factor $(i = 1, 2, \ldots, v)$ in the u-th run $(u = 1, 2, \ldots, N)$ of the experiment, ε_u's are uncorrelated random errors with mean zero and variance σ^2, is said to be *second order rotatable design* (SORD), if the variance of the estimate of $Y_u(x_1, x_2, \ldots, x_v)$ with respect to each of independent variables (x_i) is only a function of the distance $(d^2 = \sum_{i=1}^{v} x_i^2)$ of the point $(x_1, x_2, \ldots, x_v)$ from the origin (center) of the design. Such a spherical variance function for estimation of responses in the second order response surface is achieved, if the design points satisfy the following conditions (Box and Hunter [1] and Das and Narasimham [2]):

(1) $\sum x_{iu} = 0, \sum x_{iu}x_{ju} = 0, \sum x_{iu}x_{ju}^2 = 0, \sum x_{iu}x_{ju}x_{ku} = 0, \sum x_{iu}^3 = 0,$

$\sum x_{iu}x_{ju}^3 = 0, \sum x_{iu}x_{ju}x_{ku}^2 = 0, \sum x_{iu}x_{ju}x_{ku}x_{lu} = 0, \text{for } i \neq j \neq k \neq l;$

(2) (i) $\sum x_{iu}^2 = \text{constant} = N\lambda_2$; (ii) $\sum x_{iu}^4 = \text{constant} = cN\lambda_4$, for all i;

(3) $\sum x_{iu}^2 x_{ju}^2 = \text{constant} = N\lambda_4$, for $i \neq j$;

(4) $\dfrac{\lambda_4}{\lambda_2^2} > \dfrac{v}{(c + v - 1)}$;

$$(5)\quad \sum x_{iu}^4 = c\sum x_{iu}^2 x_{ju}^2; \tag{2.2}$$

where c, λ_2, and λ_4 are constants.

The variances and covariances of the estimated parameters are

$$V(\hat{b}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N\left[\lambda_4(c+v-1) - v\lambda_2^2\right]},$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\lambda_2},$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2) - (v-1)\lambda_2^2}{\lambda_4(c+v-1) - v\lambda_2^2}\right],$$

$$\mathrm{Cov}(\hat{b}_0, \hat{b}_{ii}) = \frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1) - v\lambda_2^2]},$$

$$\mathrm{Cov}(\hat{b}_{ii}, \hat{b}_{jj}) = \frac{(\lambda_2^2 - \lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1) - v\lambda_2^2]}$$ and other covariances vanish. (2.3)

The variance of the estimated response at the point $(x_{10}, x_{20}, \ldots, x_{v0})$ is

$$V(\hat{y}_0) = V(\hat{b}_0) + \left[V(\hat{b}_i) + 2\,\mathrm{cov}(\hat{b}_0, \hat{b}_{ii})\right]d^2 + V(\hat{b}_{ii})\,d^4$$
$$+ \sum x_{i0}^2 x_{j0}^2\left[V(\hat{b}_{ij}) + 2\,\mathrm{cov}(\hat{b}_{ii}, \hat{b}_{jj}) - 2V(\hat{b}_{ii})\right]. \quad (2.4)$$

The coefficient of $\sum x_{i0}^2 x_{j0}^2$ in the above Equation (2.4) is simplified to $(c-3)\sigma^2/(c-1)N\lambda_4$. A second order response surface design D is said to be a SORD, if in this design $c = 3$ and all the other conditions (2.2) to (2.3) hold.

3. SORD Using Incomplete Block Designs

In this section, the method of constructions of SORD using PBD and SUBA with two unequal block sizes of Tyagi [8] and Raghavarao [6] are explained for ready reference.

3.1. SORD using PBD

Let $(v, b, r, k_1, k_2, \ldots, k_p, \lambda)$ denote a PBD, $k = \text{Sup}(k_1, k_2, \ldots, k_p)$ and $2^{t(k)}$ denote a fractional replicate of 2^k in ± 1 levels, in which no interaction with less than five factors is confounded. $[1 - (v, b, r, k_1, k_2, \ldots, k_p, \lambda)]$ denote the design points generated from the transpose of incidence matrix of PBD. $[1 - (v, b, r, k_1, k_2, \ldots, k_p, \lambda)]$ $2^{t(k)}$ are the $b2^{t(k)}$ design points generated from PBD by "multiplication" (c.f. Raghavarao [7], 298-300), $(a, 0, 0, \ldots, 0)2^1$ denote the design points generated from $(a, 0, 0, \ldots, 0)$ point set, $\bigcup$ denotes combination of the design points generated from different sets of points, and n_0 denote the number of central points. The method of construction of SORD using PBD is given in the following result (cf. Tyagi [8]):

Result 3.1. The design points, $[1 - (v, b, r, k_1, k_2, \ldots, k_p, \lambda)]2^{t(k)} \bigcup$ $(a, 0, \ldots, 0)2^1 \bigcup (n_0)$ will give a v-dimensional SORD in $N = b2^{t(k)} + 2v$ $+ n_0$ design points, with $a^4 = (3\lambda - r)2^{t(k)-1}$.

3.2. SORD using SUBA with two unequal block sizes

Let $(v, b, r, k_1, k_2, b_1, b_2, \lambda)$ denote a SUBA with two unequal block sizes, $b_1 + b_2 = b$, $k = \text{Sup}(k_1, k_2)$ and $2^{t(k)}$ denote a fractional replicate of 2^k in ± 1 levels, in which no interaction with less than five factors is confounded. $[1 - (v, b, r, k_1, k_2, b_1, b_2, \lambda)]$ denote the design points generated from the transpose of incidence matrix of SUBA with two unequal block sizes. $[1-(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from SUBA with two unequal block sizes by "multiplication", $(a, 0, 0, \ldots, 0)2^1$ denote the design points generated from $(a, 0, 0, \ldots, 0)$ point set, $\bigcup$ denotes combination of the design points generated from different sets of points, and n_0 denote the number of central points. The method of construction of SORD using SUBA with two unequal block sizes is given in the following result (cf. Raghavarao [6]).

Result 3.2. The design points, $[1-(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)} \bigcup$ $(a, 0, \ldots, 0)2^1 \bigcup (n_0)$ will give a v-dimensional SORD in $N = b2^{t(k)} + 2v + n_0$ design points, with $a^4 = (3\lambda - r)2^{t(k)-1}$.

4. Conditions of Measure of Rotatability for Second Order Response Surface Designs

Following Box and Hunter [1], Das and Narasimham [2]; Park et al. [9], equations in 1, 2, 3, 4, 5 of (2.2) and (2.3) give the necessary and sufficient conditions for measure of rotatability for any general second order response surface designs. Further, we have

$V(b_i)$ are equal for i,

$V(b_{ii})$ are equal for i,

$V(b_{ij})$ are equal for i, j, where $i \neq j$,

$$\mathrm{Cov}(b_i, b_{ii}) = \mathrm{Cov}(b_i, b_{ij}) = \mathrm{Cov}(b_{ii}, b_{ij}) = \mathrm{Cov}(b_{ij}, b_{il}) = 0,$$

$$\text{for all } i \neq j \neq l \neq i. \tag{4.1}$$

Park et al. [9] suggested that if the conditions in (2.2) together with (2.3) and (4.1) are met, then the following measure $(P_v(D))$ given below asses the degree of measure of rotatability for any general second order response surface design (cf. Park et al. [9], page 661).

$$P_v(D) = \frac{1}{1 + R_v(D)}, \tag{4.2}$$

where

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \frac{6v[V(\hat{b}_{ij}) + 2\operatorname{cov}(\hat{b}_{ii}, \hat{b}_{jj}) - 2V(\hat{b}_{ii})]^2(v-1)}{(v+2)^2(v+4)(v+6)(v+8)g^8}, \tag{4.3}$$

and g is the scaling factor.

On simplification, numerator of (4.3), $[V(\hat{b}_{ij}) + 2\operatorname{cov}(\hat{b}_{ii}, \hat{b}_{jj}) - 2V(\hat{b}_{ii})]$ using (2.3) becomes $\frac{(c-3)\sigma^2}{(c-1)N\lambda_4}$. Thus, $R_v(D)$ becomes

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \left(\frac{6v[(c-3)\sigma^2]^2(v-1)}{((c-1)N\lambda_4)^2(v+2)^2(v+4)(v+6)(v+8)g^8}\right). \tag{4.4}$$

Note. For SORD, we have $c = 3$. Substituting the value of 'c' in (4.4) and on simplification we get $R_v(D)$ is zero. Hence from (4.2), we get $P_v(D)$ is one if and only if a design is rotatable and less than one for a non-rotatable design.

5. Measure of Rotatability for Second Order Response Surface Designs Using Incomplete Block Designs

In this section, the proposed new method of measure of rotatability for second order response surface designs using PBD and SUBA with two unequal block sizes are suggested below:

5.1. Measure of rotatability for second order response surface designs using PBD

Let $(v, b, r, k_1, k_2, \ldots, k_p, \lambda)$ denote a PBD, $k = \operatorname{Sup}(k_1, k_2, \ldots, k_p)$ and $2^{t(k)}$ denote a fractional replicate of 2^k in ± 1 levels, in which no interaction with less than five factors is confounded. $[1 - (v, b, r, k_1, k_2, \ldots, k_p, \lambda)]$ denote the design points generated from the transpose of incidence matrix of PBD. $[1-(v, b, r, k_1, k_2, \ldots, k_p, \lambda)]$ $2^{t(k)}$ are the $b2^{t(k)}$ design points generated from PBD by "multiplication" (c.f. Raghavarao [7], 298-300), $(a, 0, 0, \ldots, 0)2^1$ denote the design points generated from $(a, 0, 0, \ldots, 0)$ point set.

Theorem 5.1. *The design points,* $[1-(v, b, r, k_1, k_2, \ldots, k_p, \lambda)]2^{t(k)}$ $\bigcup (a, 0, \ldots, 0)2^1 \bigcup (n_0)$ *will give a v-dimensional measure of rotatability for second order response surface designs using PBD in* $N = b2^{t(k)} + 2v + n_0$ *design points, with level 'a' pre-fixed and* $c = \dfrac{r2^{t(k)} + 2a^4}{\lambda 2^{t(k)}}$.

Proof. For the design points generated from PBD, simple symmetry condition 1 of (2.2) are true. Further, conditions 2 and 3 of (2.2) are true as follow:

$$\sum x_{iu}^2 = r2^{t(k)} + 2a^2 = N\lambda_2, \tag{5.1}$$

$$\sum x_{iu}^4 = r2^{t(k)} + 2a^4 = cN\lambda_4, \tag{5.2}$$

$$\sum x_{iu}^2 x_{ju}^2 = \lambda 2^{t(k)} = N\lambda_4. \tag{5.3}$$

From (5.2) and (5.3), we get $c = \dfrac{r2^{t(k)} + 2a^4}{\lambda 2^{t(k)}}$. The measure of rotatability values for second order response surface designs using PBD can obtain as follow. From (4.2), we have

$$P_v(D) = \frac{1}{1 + R_v(D)},$$

where

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \left(\frac{6v((c-3)\sigma^2)^2(v-1)}{((c-1)N\lambda_4)^2(v+2)^2(v+4)(v+6)(v+8)g^8}\right),$$

and the scaling factor

$$g = \begin{cases} \frac{1}{a}, \text{ if } a \le \sqrt{2^{t(k)-1}(b-r)+v}, \\ \frac{1}{\sqrt{2^{t(k)-1}(b-r)+v}}, \text{ if } a > \sqrt{2^{t(k)-1}(b-r)+v}. \end{cases}$$

Example 5.1. We illustrate Theorem (5.1) with measure of rotatability for second order response surface design using a PBD with parameters $(v = 9,\ b = 11,\ r = 5,\ k_1 = 5,\ k_2 = 4,\ k_3 = 3,\ \lambda = 2)$ The design points $[1-(9, 11, 5, 5, 4, 3, 2)]2^{t(5)}U(a, 0, 0, \ldots, 0)2^1U(n_0 = 1)$ will give a measure rotatability for second order response surface design in $N = 195$ design points for nine factors (taking one central point). From (5.1), (5.2), and (5.3), we have

$$\sum x_{iu}^2 = 80 + 2a^2 = N\lambda_2, \tag{5.4}$$

$$\sum x_{iu}^4 = 80 + 2a^4 = cN\lambda_4, \tag{5.5}$$

$$\sum x_{iu}^2 x_{ju}^2 = 32 = N\lambda_4. \tag{5.6}$$

From (5.5) and (5.6), we can obtain the rotatability value by taking $c = 3$. Hence, we get a SORD with $a = 1.6818$ and $c = 3$ (cf. Tyagi [8]). Instead of taking $a = 1.6818$, suppose we take $a = 1.6$, we get $c = 2.9096$. The scaling factor $g = 0.1667$, $R_v(D) = 0.003959$ and $P_v(D) = 0.9961$.

Hence, we get a nearly SORD using PBD with $N = 195$, $a = 1.6$, $c = 2.9096$, scaling factor $g = 0.1667$ and measure of rotatability $P_v(D) = 0.9961$.

Tables 5.1 gives the values of measure of rotatability for second order response surface designs using PBD. It can be verify that $P_v(D)$ is 1 if and only if the design is rotatable design and it is smaller than one for a non-rotatable design.

Table 5.1. Measure of rotatability for second order response surface designs using PBD

a	(9, 11, 5, 5, 4, 3, 2) $N = 195$	(10, 11, 5, 5, 4, 2,) $N = 197$	(13, 15, 7, 7, 6, 5, 3) $N = 987$	(14, 15, 7, 7, 6, 3) $N = 989$
1.3	0.9882	0.9896	0.9719	0.9752
1.6	0.9962	0.9966	0.8874	0.8994
1.9	0.8886	0.9005	0.7356	0.7593
2.2	0.3010	0.3283	0.6103	0.6398
2.5	0.0633	0.0712	0.6447	0.6730
2.8	0.0166	0.0188	0.9892	0.9904
3.1	5.5471×10^{-3}	6.2890×10^{-3}	0.3041	0.3314
3.4	2.1995×10^{-3}	2.4947×10^{-3}	0.0465	0.0524
3.7	9.8748×10^{-4}	1.1202×10^{-3}	0.0113	0.0128
4.0	4.8611×10^{-4}	5.5151×10^{-4}	3.7518×10^{-3}	4.2529×10^{-3}
4.3	2.5683×10^{-4}	2.9139×10^{-4}	1.5071×10^{-3}	1.7089×10^{-3}
4.6	1.4352×10^{-4}	1.6283×10^{-4}	6.9232×10^{-4}	7.8510×10^{-4}
4.9	8.3942×10^{-5}	9.5239×10^{-5}	3.5072×10^{-4}	3.9774×10^{-4}
*	1.6818	1.6818	2.8284	2.8284

'$*$' indicates values of SORD using PBD.

5.2. Measure of rotatability for second order response surface designs using SUBA with two unequal block sizes

Let $(v, b, r, k_1, k_2, b_1, b_2, \lambda)$ denote a SUBA with two unequal block sizes, $b_1 + b_2 = b$, $k = \text{Sup}(k_1, k_2)$ and $2^{t(k)}$ denote a fractional replicate of 2^k in ± 1 levels, in which no interaction with less than five factors is confounded. $[1 - (v, b, r, k_1, k_2, b_1, b_2, \lambda)]$ denote the design points generated from the transpose of incidence matrix of SUBA with two unequal block sizes. $[1 - (v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$

design points generated from SUBA with two unequal block sizes by "multiplication", $(a, 0, 0, \ldots, 0)2^1$ denote the design points generated from $(a, 0, 0, \ldots, 0)$ point set.

Theorem 5.2. *The design points,* $[1-(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)} \bigcup (a, 0, \ldots, 0)2^1 \bigcup (n_0)$ *will give a v-dimensional measure of rotatability for second order response surface designs using SUBA with two unequal block sizes in* $N = b2^{t(k)} + 2v + n_0$ *design points, with level 'a' pre-fixed and* $c = \frac{r2^{t(k)} + 2a^4}{\lambda 2^{t(k)}}$.

Proof. For the design points generated from SUBA with two unequal block sizes, simple symmetry conditions are true. Further, we have

$$\sum x_{iu}^2 = r2^{t(k)} + 2a^2 = N\lambda_2, \tag{5.7}$$

$$\sum x_{iu}^4 = r2^{t(k)} + 2a^4 = cN\lambda_4, \tag{5.8}$$

$$\sum x_{iu}^2 x_{ju}^2 = \lambda 2^{t(k)} = N\lambda_4. \tag{5.9}$$

From (5.8) and (5.9), we get $c = \frac{r2^{t(k)} + 2a^4}{\lambda 2^{t(k)}}$. From (4.2), we can obtain the measure of rotatability values for second order response surface designs using SUBA with two unequal block sizes. From (4.2), we have

$$P_v(D) = \frac{1}{1 + R_v(D)},$$

where

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \left(\frac{6v((c-3)\sigma^2)^2(v-1)}{((c-1)N\lambda_4)^2(v+2)^2(v+4)(v+6)(v+8)g^8}\right),$$

and the scaling factor

$$g = \begin{cases} \dfrac{1}{a}, \text{ if } a \le \sqrt{2^{t(k)-1}(b-r)+v}, \\ \dfrac{1}{\sqrt{2^{t(k)-1}(b-r)+v}}, \text{ if } a > \sqrt{2^{t(k)-1}(b-r)+v}. \end{cases}$$

Example 5.2. We illustrate Theorem 5.2 with measure of rotatability for second order response surface design using a SUBA with two unequal sizes with parameters $(v = 6, b = 11, r = 7, k_1 = 3, k_2 = 4, b_1 = 2, b_2 = 9, \lambda = 4)$.

The design points $[1-(6, 11, 7, 3, 4, 2, 9, 4)]2^{t(4)} \bigcup (a, 0, 0, \ldots, 0)2^1 \bigcup (n_0 = 1)$ will give a measure rotatability for second order response surface design in $N = 189$ design points for six factors (taking one central point). From (5.1), (5.2) and (5.3), we have

$$\sum x_{iu}^2 = 112 + 2a^2 = N\lambda_2, \tag{5.10}$$

$$\sum x_{iu}^4 = 112 + 2a^4 = cN\lambda_4, \tag{5.11}$$

$$\sum x_{iu}^2 x_{ju}^2 = 64 = N\lambda_4. \tag{5.12}$$

From (5.11) and (5.12), we can obtain the rotatability value by taking $c = 3$. Hence, we get a SORD with $a = 2.5149$ and $c = 3$ (cf. Raghavarao [6]). Instead of taking $a = 2.5149$, suppose we take $a = 2.50$, we get $c = 2.9707$. The scaling factor $g = 0.40$, $R_v(D) = 0.004923$, and $P_v(D) = 0.9951$.

Hence, we get a nearly SORD using SUBA with two unequal block sizes with $N = 189$, $a = 2.50$, $c = 2.9707$, $g = 0.40$ and measure of rotatability $P_v(D) = 0.9951$.

Table 5.2 gives the values of measure of rotatability for second order response surface designs using SUBA with two unequal block sizes. It can be verify that $P_v(D)$ is 1 if and only if the design is rotatable design and it is smaller than one for a non-rotatable design.

Table 5.2. Measure of rotatability for second order response surface designs using SUBA with two unequal block sizes

α	(6, 11, 7, 3, 4, 2, 9, 4) $N = 189$	(9, 12, 7, 3, 6, 3, 9, 4) $N = 403$	(10, 11, 5, 4, 5, 5, 6, 2) $N = 197$	(12, 15, 7, 4, 6, 3, 12, 3) $N = 505$
1.3	0.8145	0.8331	0.9896	0.9708
1.6	0.5710	0.5461	0.9966	0.9022
1.9	0.4320	0.3142	0.9005	0.8334
2.2	0.5054	0.2270	0.3283	0.9071
2.5	0.9951	0.2173	0.0712	0.8779
2.8	0.2232	0.4638	0.0188	0.1982
3.1	0.0382	0.5864	6.2890×10^{-3}	0.0390
3.4	0.0108	0.0567	2.4947×10^{-3}	0.0108
3.7	3.9947×10^{-3}	0.0126	1.1202×10^{-3}	3.8232×10^{-3}
4.0	1.7532×10^{-3}	4.2461×10^{-3}	5.5151×10^{-4}	1.6016×10^{-3}
4.3	8.6008×10^{-4}	1.7731×10^{-3}	2.9139×10^{-4}	7.5552×10^{-4}
4.6	4.5712×10^{-4}	8.4837×10^{-4}	1.6283×10^{-4}	3.8917×10^{-4}
4.9	2.5814×10^{-4}	4.4572×10^{-4}	9.5239×10^{-5}	2.1444×10^{-4}
*	2.5149	2.9907	1.6818	2.3784

'*' indicates values of SORD using SUBA with two unequal block sizes.

6. Conclusion

In this paper, a new measure of rotatability for second order response surface designs using incomplete block designs have been proposed, which enables us to assess the degree of rotatability for a given second order response surface design.

It may be pointed out here that the measure of rotatability for second order response surface designs using PBD with parameters $(v = 9, b = 11, r = 5, k_1 = 4, k_2 = 4, k_3 = 3, \lambda = 2)$ has only 195 design points for 9-factors, where as the corresponding measure of rotatability for second order response surface design using BIBD with parameters $(v = 9, b = 18, r = 8, k = 4, \lambda = 3)$ of Victorbabu and Surekha [10] needs 307 design points. Similarly, the measures for second order response surface design using PBD and SUBA with two unequal block sizes have only 197 design points for 10-factors, whereas the corresponding measure for second order response surface design using BIBD $(v = 10, b = 18, r = 9, k = 5, \lambda = 4)$ needs 309 design points. For 12-factors, the measure of rotatability for second order response surface design using SUBA with two unequal block sizes has only 505 design points, whereas the corresponding the measure of rotatability for second order response surface design using BIB design $(v = 12, b = 22, r = 11, k = 6, \lambda = 5)$ needs 729 design points. Thus, these new methods sometimes lead to the measure of rotatability for second order response surface design using PBD and SUBA with two unequal block sizes with lesser number of design points than the measure of rotatability for second order response surface designs using BIBD.

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